Letters

Comments on "General Oscillator Characterization Using Linear Open-Loop S-Parameters"

Lucia Cascio

I. Introduction

The above paper[1] presents an expression for the evaluation of the open-loop gain of the transmission model of a generic oscillator in terms of its S-parameters. As correctly stated by Randall and Hock, the virtual ground technique leading to the transmission model was first introduced by Alechno in [1], where it was assumed that the open-loop gain was simply given by S_{21}. Unfortunately, Randall and Hock do not cite another important paper [2] successively published on this topic, where the open-loop gain has been determined in the case of unilateral devices. In Harada's paper, the gain expression was correctly determined as

$$G = \frac{S_{21}}{1 - S_{11} \cdot S_{22}}. \tag{1}$$

Since, in most circumstances, it is acceptable to assume that the active device is unilateral (as also stated in the above paper), (1) constitutes an excellent approximation to determine the oscillation condition based on the S-parameters of the feedback oscillator.

In the most general case of devices with $S_{12} \neq 0$, Randall and Hock present two different formulations for the determination of the open-loop gain, based, respectively, on the T (transfer scattering) parameters and on the S-parameters of the open-loop circuit

$$G(T) = \frac{2}{(T_{11} + T_{22}) \pm \sqrt{(T_{11} + T_{22})^2 - 4\,(T_{11}T_{22} - T_{12}T_{21})}} \tag{2}$$

$$G(S) = \frac{S_{21} - S_{12}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}}. \tag{3}$$

In Randall and Hock's approach, they seem to derive (3) from (2), by first substituting the T_{ij} terms with equivalent Z-parameter expressions, assuming that no current will flow in a lumped Z" element of the circuit, and finally expressing the Z-parameter condition in terms of the S-parameters. Unfortunately, (2) and (3) do not yield the same results, thus invalidating one or the other. The procedure illustrated by Randall and Hock alters the load effect due to Z", thus affecting the circuit input and output impedances.

In the following, a new equation for the open-loop gain based on the scattering parameters is presented, which is equivalent to that of the T-parameters published in the above paper. The correct equation is based on the determination of the input reflection coefficient $\Gamma_{\rm in}$ of the self-terminated model.

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[1]M. Randall and T. Hock, *IEEE Trans. Microwave Theory Tech.*, vol. 49, no. 6, pp. 1094–1100, June 2001.

II. New Generalized Gain Expression in Terms of S-parameters

The most natural way to derive the oscillation condition for the open-loop transmission model is to consider a finite series [3] or, ideally, an infinite cascade of identical networks, as in Fig. 3 of the above paper.

In this case, each section is loaded at its output by an identical impedance Z_L, thereby causing identical reflection coefficients. On the other hand, the output load of each section is also the input port of the successive network, having input impedance $Z_{\rm in}$. Therefore, the condition $Z_L = Z_{\rm in}$ must be satisfied for each section. From basic microwave theory [4], this condition yields the following solutions for $\Gamma_{\rm in}$:

$$\Gamma_{\rm in} = \frac{1+S_{11}\cdot S_{22}-S_{12}\cdot S_{21} \pm \sqrt{(1+S_{11}\cdot S_{22}-S_{12}\cdot S_{21})^2 - 4\cdot S_{11}\cdot S_{22}}}{2\cdot S_{22}}. \tag{4}$$

The expression for $\Gamma_{\rm out}$ can be obtained in a similar way, and is the same as (4), except for S_{11} replacing S_{22} in the denominator.

It is interesting to observe that since $b_2 = a_1$ and $a_2 = b_1$, for the circuit to oscillate, the condition $\Gamma_{\rm in} = 1/\Gamma_{\rm out}$ must be satisfied. Therefore, either the input or output impedance of the transmission model actually has a negative resistance. This is the same type of condition imposed for the analysis of negative resistance oscillators; the feedback oscillator can then be seen as a more general case of a negative resistance oscillator, where S_{12} and $S_{21} \neq 0$.

The open-loop gain is then derived by calculating the ratio of the total voltages V_1 and V_2. This ensures that the gain expression does not depend on the characteristic impedances (Z_{01}, Z_{02}) used to define the S-parameters

$$G(S) = \frac{V_2}{V_1} = \frac{\sqrt{Z_{02}}}{\sqrt{Z_{01}}} \frac{S_{21}}{1 - S_{22} \cdot \Gamma_{\rm in}}. \tag{5}$$

Assuming that $Z_{01} = Z_{02}$ and substituting (4) for $\Gamma_{\rm in}$, it can be shown that (5) and (2) are identical. In particular, the minus sign in the expression for $\Gamma_{\rm in}$ corresponds to the solution with positive exponential growth in (2) and vice versa. The advantage of this formulation (5) is that it is directly expressed in terms of the S-parameters, rather than T-parameters, and is related to the physical concept of the open-loop input impedance.

III. Other Comments

Randall and Hock also indicate that the positive sign should be chosen in (2). While this is true in proximity of the oscillation point for $S_{12} \ll 1$, in general, the sign must be chosen so that the continuity of the expression is preserved, especially when dealing with wide-bandwidth S-parameters. This might require switching from one sign to the other across the bandwidth of interest.

Finally, the last sentence of paragraph 4 on page 1095 of the above paper should read: "In that case, S_{12} and S_{21} are zero and the equation reduces to the familiar form $S_{11}S_{22} = 1$."

IV. Conclusion

In conclusion, a general expression for the gain in terms of S-parameters, based on the physical concept of the input and output impedances of the self-terminated circuit, has been provided. This expression is mathematically equivalent to the one provided in terms of T-parameters in the above paper, and invalidates the S-parameter formulation provided by Randall and Hock.

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References

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Authors' Reply

Mitch Randall and Terry Hock

Cascio's comments and S-parameter derivation touch on an important fundamental concept underlying the derivation described in the above paper.[1] The above paper did not go into depth explaining some of the subtleties related to this, but we welcome the opportunity to do so now.

In the above paper, we consider an infinite series of identical networks with the goal of accounting for the impedance mismatch when a single network is self-connected. However, care has to be taken to interpret and use this concept appropriately since an infinite series of networks is *not* the same as a single self-connected network.

In particular, consider a well-designed single-ended open-loop oscillator network Z. To this network we add a resistor R to a new single-ended ground defining a new impedance matrix $\tilde{Z}$, as shown in Fig. 1, so that

$$\tilde{Z} = \begin{bmatrix} z_i + R & z_r + R \\ z_f + R & z_o + R \end{bmatrix}. \quad (1)$$

It is self evident that we have changed nothing fundamental about the operation of the oscillator when self-connected; its operation frequency, loop gain, startup time, loaded Q, etc, remain unchanged. When self-connected, no current flows through the added resistance.

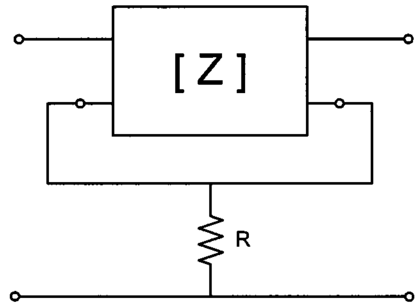

Fig. 1. Resistor R added to an oscillator network as shown changes nothing about its closed-loop performance. The expression characterizing oscillator performance should not depend on this resistor.

However, we *have* changed the open-loop two-port network and its response to various steady-state test signals.

When an infinite series of the new networks $\tilde{Z}$ are cascaded and a steady-state test signal is applied, current will, in general, flow through the resistance R. For example, as the resistance R increases to very large values $(R \gg \tilde{z}_i)$, the new network input impedance approaches $\tilde{z}_i = R$ and the "gain" $\tilde{G}$ approaches unity, as can be shown from the Z-parameter expression

$$\lim_{R\to\infty} \tilde{g} = \frac{\tilde{z}_f}{\tilde{z}_i + \tilde{z}_o} = \frac{z_f + R}{z_i + z_o + 2R} = \frac{1}{2}. \quad (2)$$

Thus,

$$\begin{aligned} \lim_{R\to\infty} \tilde{G} &= \lim_{R\to\infty} \frac{2\tilde{g}}{1 + \sqrt{1 - 4\frac{\tilde{z}_r}{\tilde{z}_f}\tilde{g}^2}} \\ &= \lim_{R\to\infty} \frac{2\left(\frac{1}{2}\right)}{1 + \sqrt{1 - 4\frac{z_r + R}{z_f + R}\left(\frac{1}{2}\right)^2}} \\ &= 1. \end{aligned} \quad (3)$$

This does not invalidate the approach since, when each network is initialized with identical exponentially growing sine waves at the frequency of oscillation, no current flows through the resistors. What is needed is a trick to remove the effect of the added resistor in our *steady-state* analysis while preserving the portion of the impedances that affect the oscillations. To do this, we consider a general single-ended open-loop oscillator network Z. The network can be represented using an ideal unilateral voltage amplifier, as shown in Fig. 2. When the circuit is drawn in this way, it is easy to see that the element z_r plays no role in the oscillations and, therefore, can be removed from consideration so that the steady-state gain takes on an unambiguous meaning. This step is described in the above paper and removes not only the added resistance R inserted above, but also any additional series impedance component already present (but perhaps not obvious) in the original network. No such series impedances, whether added externally or intrinsic to the network, are involved in the oscillations. In order to get meaningful and unambiguous results from a steady-state analysis, all of these elements must be removed. The remaining network containing only those elements that are involved in the oscillations is characterized by

$$Z' = \begin{bmatrix} z_i - z_r & 0 \\ z_f - z_r & z_o - z_r \end{bmatrix}. \quad (4)$$

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